International Conference on Control, Engineering & Information Technology (CEIT'13)
Proceedings Engineering & Technology - Vol.4, pp. 67-72, 2013

Particle Filter for State and Unknown Input Estimation of Chaotic Systems

Sameh Mejri, Ali Sghaier Tlili and Naceur Benhadj Braiek
Laboratoire de Systèmes Avancés (LSA)
Ecole Polytechnique de Tunisie, BP. 743, 2078, La Marsa, Tunisie
Université de Carthage
E-mail: sameh3000@hotmail.fr, ali.tlili@ept.rnu.tn, naceur.benhadj@ept.rnu.tn

***Abstract*—Chaotic systems exhibit highly nonlinear, complex and unpredictable behaviors. Thereby, these systems have received much attention in a variety of fields over the past few decades. In this paper we develop a particle filter algorithm to solve the problem of chaotic state and unknown input estimation from arbitrarily nonlinear time series. Thus, even if there exist Gaussian or non-Gaussian noise in chaotic maps, not only the chaotic states can be estimated by the proposed particle filter but also the unknown inputs.**

A computer simulation is conducted on the famous Holmes map to demonstrate the effectiveness and the high performances of the proposed estimation approach.

I. Introduction

In the past decades chaos has been an interesting topic in the field of nonlinear science [1], [2]. Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions which is popularly referred to as the butterfly effect [3], [4]. Small differences in initial conditions (such as those due to rounding errors in numerical computation) yield widely diverging outcomes for such dynamical systems, rendering estimation more difficult [5], [6].

Among many studies on chaos, one important topic is to estimate chaotic states from the time series deriving from chaotic maps [7], [8]. Many works considered the estimation of chaotic states with the assumptions that chaotic maps and their time series are only affected by white noise, in particular Gaussian noise, the time series is linear on chaotic states, and there exist no unknown inputs in chaotic maps.

Note that estimating unknown inputs is motivated by certain applications such as fault detection, fault diagnostic, control system design and synchronization and decryption in chaotic communication systems. Filter design for estimating the state of a system and the unknown inputs in the linear case has received considerable attention in the past. However, little research has been paid toward nonlinear case [9].

Additionally, most of the existing results for nonlinear systems concerns only the estimation of the system state subject to unknown inputs, see Chen and Saif [10], Pertew et al. [11] for instance. Very few works have been carried out on estimating the unknown inputs. We cite here some of the available results in input recovery context. In Huijberts et al. [12], the problem of unknown, constant or slowly time-varying input estimation using an Extended Kalman Filter (EKF) is discussed. In Boutayeb et al. [13], the authors proposed an approach to estimate simultaneously the state of the system and the unknown inputs using a generalized state space observer. This approach is extended recently to a more general class of nonlinear systems in Trinh et al. [14], and to discrete time case using the EKF in Boutayeb [15].

In certain applications, such as chaos communication, the quality of the input reconstruction plays a very important role. Indeed, in chaos communication systems, the transmitted signal through a transmission channel is often corrupted by noise. Therefore, proposing a new estimation unknown input method that take into account the noises affecting the systems is necessary.

In this paper, the particle filtering technique is proposed to solve the problem of state and the input estimation of chaotic systems in the presence of Gaussian or non-Gaussian noise. Indeed, the first particle filter algorithm was proposed by Gordon [16]. After that, a number of particle filter algorithms have been proposed such as auxiliary sampling importance resampling particle filter [17] and regularized particle filter [18], [19]. Following Bayesian filtering framework, particle filters use sequential Monte Carlo methods to approximate the optimal filtering by representing the probability density function with a swarm of particles [20].

On the other hand, particle filter algorithms consist of two main steps, namely prediction and update which enable particle filters to perform online estimation recursively. Moreover, particle filters have been successfully applied to many scientific and engineering fields such as tracking problems [21], speech enhancement [22], [23], fault detection [24], [25], recovering the hidden messages in the field of chaotic secure communication [26], [27].

The main contribution of this paper lies in the extension of particle filter to unknown input reconstruction using an approximate Bayesian classifier and its application to the estimation of chaotic systems. Thus, the proposed particle filter estimates not only the chaotic state but also reconstructs the unknown input from arbitrarily nonlinear time series even if exist Gaussian or non-Gaussian noise in chaotic maps. In this situation, both the efficiency and the robustness of the proposed algorithm can be improved.

The organization of this paper is as follows. In Section II, some preliminaries of this paper are reviewed. In Section III, an estimation method based on particle filters is proposed for the chaotic state estimation and unknown input reconstruction despite the presence of either Gaussian or non-Gaussian noise. In Section IV, numerical simulations on the Holmes process are given to demonstrate the effectiveness of the proposed estimation approach. Finally, some concluding remarks are provided in Section V.

II. Preliminary fundamentals and problem formulation

A. Overview of particle filters

The particle filter is also referred to as sequential Monte Carlo approach in the Bayesian framework. It does not estimate the state information explicitly but with a posterior probability. This technique can achieve theoretically optimal solution for nonlinear/non-Gaussian models.

Let us consider a dynamic system represented by :

$$\begin{cases} x_{k+1} &= f(x_k) + r_k \\ y_k &= h(x_k) + v_k \end{cases} \quad (1)$$

where $x_k \in \Re^n$ is the state vector and $y_k \in \Re^m$ is the output vector. $f(.)$ and $h(.)$ are the system and measurement equation, respectively, and r_k and v_k are the system and measurement noises, respectively.

In order to represent the posterior probability $p(x_k/Y_k)$, we define the particle set at time k as follows:

$$P_k = \left\{(x_k^i, w_k^i)\backslash i = 1, ..., N\right\} \quad (2)$$

where $x_k^i \in \Re^n$ denotes the ith particle of P_k and w_k^i is the associated importance weight.

By using the set of particles, we approximate the posterior probability $p(x_k/Y_k)$ as :

$$p(x_k/Y_k) = \sum_{i=1}^{N} w_k^i \delta(x_k - x_k^i) \quad (3)$$

where $Y_k = \{y_1, y_2, ..., y_k\}$ is the set of accumulated measurements up to time k.

In order to estimate the state, the particle filter performs three steps at each time which are sampling, importance weighting and resampling.

Indeed, in the sampling step, samples are generated according to a proposal distribution $q(x_k^i/x_{k-1}^i, y_k)$. The choice of the proposal density is one of the most critical issues in the particle filter and two popular choices are the posterior function $q(x_k^i/x_{k-1}^i, y_k) = p(x_k^i/x_{k-1}^i, y_k)$ and the prior function $q(x_k^i/x_{k-1}^i, y_k) = p(x_k^i/x_{k-1}^i)$ [19].

In the importance weighting process, the importance weights are updated such the set of weighted particles approximate the unknown target distribution.

At the resampling step, the particles are resampled according to their weights and are replaced with new particles with equal weights given by $1/N$ [28], [29].

Remark 1: It is noteworthy that the particles with larger weights are always more likely to be selected than the particles with smaller weights in order to obtain good estimation performances.

B. Problem formulation

Consider chaotic maps with unknown input given as follows:

$$\begin{cases} x_{k+1} &= f(x_k) + d + r_k \\ y_k &= h(x_k) + v_k \end{cases} \quad (4)$$

where $x_k \in R^n$ is the state vector, $y_k \in R^p$ is the output vector, $f(x_k)$ stands for the nonlinear function and the function $h(x_k)$, considered in the time series y_k, has an arbitrarily nonlinear form. $r_k \in R^n$ and $v_k \in R^p$ represent the system noise and the measurement noise, respectively. The constant vector $d \in R^n$ is regarded as the unknown input to be estimated.

Remark 2: Notice that the unknown input d does not change the chaos nature. Furthermore, the system noise r_k and the measurement noise v_k could be either Gaussian or non-Gaussian noise.

Our problem undertaken in this paper is to estimate the chaotic states and to reconstruct the unknown input from the nonlinear output time series despite the presence of either Gaussian or non-Gaussian noise applied to the chaotic system. Hence, in the subsequent sections, the particle filter algorithm design is proposed to resolve the above problem.

III. Particle filter algorithm with unknown input reconstruction

In this section, we first introduce the unknown input reconstruction with an approximate Bayesian classifier. Then, we present the proposed particle filter algorithm in order to estimate simultaneously the chaotic state variables and the unknown inputs.

A. Unknown input reconstruction with a Bayesian classifier

We note that the constant vector d must take a value belongs to the set $S = \{\theta_1, \theta_2, ..., \theta_M\}$ where the parameters $\theta_1, \theta_2, ..., \theta_M$ are different vectors which do not change the chaos nature of the studied process.

The probability of each element of the set S, denoted by $P\left(d_k = \theta_l\right)$ for $l = 1, 2..., M$, is assumed to be known as prior knowledge.

On receiving the measurement y_{k+1}, according to Bayesian formula [30], we have :

$$P\left(d_k=\theta_l/y_{k+1}\right)=\frac{p\left(y_{k+1}/d_k=\theta_l\right)P\left(d_k=\theta_l\right)}{\sum_{j=1}^{M}p\left(y_{k+1}/d_k=\theta_j\right)P\left(d_k=\theta_j\right)} \tag{5}$$

for $l=1,2...,M$.

Then, we approximate the prior probability density $p\left(y_{k+1}/d_k=\theta_l\right)$ by Monte Carlo method [16] :

$$\begin{aligned} p\left(y_{k+1}/d_k=\theta_l\right) &= \int p_v\left(y_{k+1}-h\left(x_{k+1/k}+\theta_l\right)\right)dx_{k+1/k} \\ &\approx \frac{1}{Np}\sum_{i=1}^{N}p_v\left(y_{k+1}-h\left(x_{k+1/k}^i+\theta_l\right)\right) \end{aligned} \tag{6}$$

for $l=1,2...,M$, where $p_v(v)$ is the probability density function of the measurement noise and N is the number of particles used in numerical simulation. Hence, the posterior probability can be approximated by :

$$\begin{aligned} P\left(d_k=\theta_l/y_{k+1}\right) &= \frac{p\left(y_{k+1}/d_k=\theta_l\right)P\left(d_k=\theta_l\right)}{\sum_{j=1}^{M}p\left(y_{k+1}/d_k=\theta_j\right)P\left(d_k=\theta_j\right)} \\ \approx & \frac{\frac{1}{Np}\sum_{i=l}^{N}p_v\left(y_{k+1}-h\left(x_{k+1/k}^i+\theta_l\right)\right)P\left(d_k=\theta_l\right)}{\frac{1}{Np}\sum_{j=l}^{M}\sum_{i=1}^{N}p_v\left(y_{k+1}-h\left(x_{k+1/k}^i+\theta_j\right)\right)P\left(d_k=\theta_j\right)} \\ &\propto \sum_{i=1}^{N}p_v\left(y_{k+1}-h\left(x_{k+1/k}^i+\theta_l\right)\right)P\left(d_k=\theta_l\right) \end{aligned} \tag{7}$$

for $l=1,2...,M$.

An approximate Bayesian classifier is formed to estimate the unknown input d_k at the kth step by :

$$d'_k=\underset{\theta_l,l=1,2...,M}{\arg\max}\sum_{i=1}^{N}p_v\left(y_{k+1}-h\left(x_{k+1/k}^i+\theta_l\right)\right)P\left(d_k=\theta_l\right) \tag{8}$$

Finally, the unknown input is defined by the following equality:

$$d'=\frac{1}{k}\sum_{i=1}^{k}d'_k \tag{9}$$

B. *Proposed particle filter algorithm*

In the subsequent section, we present a proposed particle filter algorithm to estimate the chaotic state variables and to reconstruct the unknown inputs applied to nonlinear chaotic systems. Thus, we introduce a few necessary improvements on the original particle filter algorithm proposed in [16].

Let D_k=$\{y_i : i=1,...,k\}$ denotes the available information of the measurement signal at the kth step. We summarize the proposed particle filter algorithm in the following algorithm:

- **Step** 1: *Initialization*
  Sampling N particles $\left\{x_0^i, i=1,...,N\right\}$ from the supposed conditional probability density function given by $p(x_0/D_0)$;

- **Step** 2: *Prediction*
  Sampling N values $\left\{r_k^i, i=1,...,N\right\}$ from the probability density function of r_k.
  Then calculate :
  $$x_{k+1/k}^i=f(x_k^i)+r_k^i \tag{10}$$
  Therefore, the prior probability function of x_{k+1} at time step k is approximated by :
  $$p(x_{k+1}/D_k)=\frac{1}{N}\sum_{i=1}^{N}\delta(x_{k+1/k}-x_{k+1/k}^i) \tag{11}$$
  where δ is the Dirac-delta function;

- **Step** 3: *Estimation*
  On receiving the measurement signal y_{k+1}, the approximate Bayesian classifier is formed to estimate the unknown input d_k at the kth step by :
  $$d'_k=\underset{\theta_l,l=1,2...,M}{\arg\max}\sum_{i=1}^{N}p_v\left(y_{k+1}-h\left(x_{k+1/k}^i+\theta_l\right)\right)\times P\left(d_k=\theta_l\right) \tag{12}$$
  Then calculate :
  $$\tilde{x}_{k+1/k}^i=f(x_k^i)+d'_k+r_k^i \tag{13}$$

- **Step** 4: *Update*
  Calculate the weight w^i by :
  $$w^i=\frac{p_v(y_{k+1}-h(\tilde{x}_{k+1/k}^i))\delta(x_{k+1/k}-\tilde{x}_{k+1/k}^i)}{\sum_{j=1}^{N}p_v(y_{k+1}-h(\tilde{x}_{k+1/k}^j))\delta(x_{k+1/k}-\tilde{x}_{k+1/k}^j)} \tag{14}$$
  then the posterior probability density function is approximated as :
  $$p(x_{k+1}/D_{k+1})=\sum_{i=1}^{N}w^i\delta(x_{k+1}-\tilde{x}_{k+1/k}^i) \tag{15}$$

- **Step** 5: *Resampling*
  Resample independently N times from the above discrete distribution. Thus, the updated probability density function becomes :
  $$p(x_{k+1}/D_{k+1})=\frac{1}{N}\sum_{i=1}^{N}\delta(x_{k+1}-x_{k+1}^i) \tag{16}$$

- **Step** 6: *Iteration*
  Let $k=k+1$, go to step 2;

- **Step** 7: *Result*
  Calculate the estimated unknown input by the following equality:
  $$d'=\frac{1}{k}\sum_{i=1}^{k}d'_k \tag{17}$$

IV. Numerical simulation

In this section, the famous Holmes map is illustrated to verify the effectiveness of the proposed particle filter algorithm in the field of chaotic state estimation and unknown input reconstruction.

The dynamics of Holmes map with unknown input is described by [31]:

$$\begin{cases} x_{1,k+1} = x_{2,k} + \; r_{1,k} \\ x_{2,k+1} = \; a\,x_{1,k} + \; b\,x_{2,k} - \; c\,x_{2,k}^3 + \; d + \; r_{2,k} \end{cases} \tag{18}$$

where $a = 0.047$, $b = 2.4$ and $c = 0.155$.

In this example the unknown input d is applied to the second equation and $r_k = [r_{1,k} \quad r_{2,k}]^T$ represent the system noise.

In order to validate the designed particle filter algorithm, the following two cases are considered where the measurement signals are nonlinear and the noises r_k can be Gaussian or non-Gaussian.

Case 1: We assume that the system noise r_k is zero-mean Gaussian white noise with the following covariance matrix:

$$R = \begin{bmatrix} 0.0025 & 0 \\ 0 & 0.0025 \end{bmatrix}$$

We assume that the unknown input is $d = 0.4$ and the initial condition of chaotic map is characterized by $x_{1,1}(0) = x_{2,1}(0) = 0$.

On the other hand, the time series is given by $y_k = x_{1,k}\,x_{2,k}^2 + v_k$ where the measurement noise v_k is zero mean Gaussian white noise with covariance $Q = 0.01$.

In this case, the initial states for the proposed particle filter algorithm are chosen as $x_{1,1}(0) = -0.5$ and $x_{2,1}(0) = 0.5$.

The particle filter was run for different number of particles ranging between 1000 and 3000. The number of efficient particles used in the resampling procedure was set to $N = 2000$. Notice that the number of particles that gives the optimal trade-off between the estimation error vector and the run-time of the particle filter can be defined by using trials mechanism. Moreover, it should be noted that the initial particles are distributed randomly in the objective to get optimal solution.

The simulation results are plotted in Figs. 1-3. Thus, Figs. 1 and 2 are the plots of the true and estimated states of Holmes map (18), while Fig. 3 shows the true and estimated unknown input, with the following numerical value extracted from simulation:

$$d' = \frac{1}{k}\sum\nolimits_{i=1}^{k} d'_k = 0.3663$$

From these figures, the proposed particle filter algorithm can dynamically estimate not only the states of Holmes map but also the unknown input with good performances.

Case 2: We assume that the system noise r_k satisfies the following probability density function :

$$p_{rk}(r_{1,k}, r_{2,k}) = 2500e^{-100(|r_{1,k}|+|r_{2,k}|)},\; r_{2,k} \in (-\infty, +\infty) \tag{19}$$

Moreover, we consider that the unknown input is $d = 0.45$ and the initial conditions of the studied chaotic map are given by $x_{1,1}(0) = 0$ and $x_{2,1}(0) = 0$.

The time series, related to (18), is given by $y_k = |x_{2,k}|\,x_{2,k} + v_k$ where the measurement noise v_k is zero-mean Gaussian white noise with covariance value $Q = 0.01$.

In this case, the initial states for the proposed particle filter algorithm are chosen as $x_{1,1}(0) = -0.5$ and $x_{2,1}(0) = 0.5$.

As in case 1, the number of particles is taken in this case such as $N = 2000$ and the initial particles are distributed randomly in order to obtain optimal performances.

The performances of the proposed approach are shown in Fig. 4-5, in which are simulated the evolution of the real states and their observed ones of the studied Holmes nonlinear stochastic system in the presence of non-Gaussian noise.

It is clear from these curves that the proposed nonlinear chaotic estimation is efficient. Indeed, it allows the state variables to reach the real trajectories of the Holmes map despite the strong disturbances emerging on the studied system.

On the other hand, Fig. 6 shows the true and estimated unknown input applied to the second equation of the nonlinear chaotic system. The approximate bayesian classifier of the unknown input is computed as:

$$d' = \frac{1}{k}\sum\nolimits_{i=1}^{k} d'_k = 0.4139$$

It can be seen from this curve and from the mean value of the unknown input vector that the proposed particle filter permits the reconstruction of the unknown input with good performances.

From the simulation results, the proposed estimation approach, in the two cases with Gaussian or non-Gaussian noise, is reliable since the estimated states track the real states accurately. Moreover, the improved particle filter made possible a convincing estimation of the unknown input applied to the Holmes chaotic process.

V. CONCLUSION

In this paper, we have developed a particle filter algorithm for nonlinear chaotic systems by using an approximate Bayesian classifier.

The proposed approach estimates not only the chaotic states but also the unknown inputs from arbitrarily nonlinear time series in the presence of either Gaussian or non-Gaussian noise.

It has been shown from the simulation results that the proposed estimation scheme based on particle filter is efficient as it allows good reconstruction of the unavailable state variables and unknown inputs of the Holmes map chaotic system despite the perturbation noise applied to the studied process.

REFERENCES

[1] Carroll T.L., Pecora L.M. Synchronizing chaotic circuits. IEEE Trans Circ Syst (1993); 38:453-6.

[2] Wu C.W., Chua L.O. A unified framework for synchronization an control of dynamical systems. Int. J. Bifurcat. Chaos (1994); 4:979-98.

[3] Chen M.Y., Zhou D.H., Shang Y. Nonlinear feedback control of Lorenz system. Chaos, Solitons Fractals (2004); 21:295-304.

[4] Chen M., Wang C., Zhang H. Chaotic Synchronization of Exponent Stability Based on Nonlinear State Observer. Proceedings of the IEEE International Conference on Mechatronics and Automation. August 5-8, (2007); Harbin, China.

[5] Torres L., Besanc G., Georges D., Verde C. Exponential nonlinear observer for parametric identification and synchronization of chaotic systems. Mathematics and Computers in Simulation 82 (2012); 836-846.

[6] Han X.H., Chang X.M. Chaotic secure communication based on a gravitational search algorithm filter. Engineering Applications of Artificial Intelligence 25, (2012); 766-774.

[7] Kevin J. Chaotic-time-series reconstruction by the Bayesian paradigm: right results by wrong methods. Phys. Rev. E (2003); 67:026212.

[8] Freking A., Kinzel W, Kanter I. Learning and predicting time series by neural networks. Phys. Rev. E. (2002); 65:050903.

[9] Gerasimos G.R. Particle and Kalman filtering for state estimation and control of DC motors. ISA Tranactions 48 (2009); 62-72.

[10] Chen W., Saif M. Unknown input observer design for a class of nonlinear systems : an LMI approach. In 2006 American Control Conference ACC 06, Minneapolis, Minnesota, USA, June (2006).

[11] Pertew A.M., Marquez H.J., Zhao Q. h_∞ synthesis of unknown input observers for nonlinear Lipschitz systems. International Journal of Control, 78(15), (2005); 1155-1165.

[12] Huijberts H., Nijmeijer H., Willems R. System identification in communication with chaotic systems. IEEE Trans. Circuits Syst. I, 47(6), (2000).

[13] Boutayeb M., Darouach M., Rafaralahy H. Generalized state-space observers for chaotic synchronization and secure communication. IEEE Trans. Circuit. Syst. I, 49(3), (2002).

[14] Trinh H., Aldeen M., Nahavandi S. An observer design procedure for a class of nonlinear time-delay systems. Computers and Electrical Engineering, (2004); 30:61-71.

[15] Boutayeb M. Synchronization and input recovery in digital non-linear systems. IEEE Trans. Circuits Syst. II:Express Brief, 51(8), (2004)

[16] Gordon N.J., Salmond D.J., Smith A. Novel approach to nonlinear/non-Gaussian Bayesian state estimation. IEE ProcF (1993); 140:107-13.

[17] Pitt M., Shephard N. Filtering via simulation: auxiliary particle filters. J Am Statist Assoc (1999); 94:590-9.

[18] Musso C., Oudjane N., Legland F. Improving regularised particle filters. In: Doucet A, de Freitas JFG, Gordon NJ, editors. Sequential Monte Carlo methods in practice. New York: Springer-Verlag; (2001).

[19] Arulampalam M., Maskell S., Gordon N., Clapp T. A tutorial on particle filters for on-line non-linear/non-gaussian bayesian tracking, IEEE Transactions on Signal Processing 50 (2002); 174-188.

[20] Smith R., Hussain M.S. Genetic Algorithm Sequentiel Monte Carlo Methods for Stochastic Volatility and Parameter Estimation. Procedings of the Word Congress on Engineering. July 4-6, (2012); London, U.K.

[21] Gustafsson F. Particle filter theory and practice with positioning applications. Aerospace and Electronic Systems Magazine, IEEE. July (2010).

[22] Vermaak J., Andrieu C., Doucet A., et al. Particle methods for Bayesian modelling and enhancement of speech signals. IEEE Trans Speech Audio Process (2002); 10:173-85.

[23] Sunho P., Seungjin C. A constrained sequential EM algorithm for speech enhancement. Neural Networks 21 (2008); 1401-1409

[24] Li P., Kadirkamanathan V. Particle filtering based likelihood ratio approach to fault diagnosis in nonlinear stochastic systems. IEEE Trans. Syst. Man Cybern. Part. CAppl. Rev. (2001); 31:337-43.

[25] Zhang B. A Probabilistic Fault Detection Approach: Application to Bearing Fault Detection. Industrial Electronics, IEEE Transactions on May (2011).

[26] Zhang B., Chen M., Zhou D. Chaotic secure communication based on particle filtering. Chaos, Solitons and Fractals 30 (2006); 1273-1280.

[27] Park S., Hwang J.P., Kim E. A new state estimation method for chaotic signals: Map-particle filter method. Expert Systems with Applications 38 (2011); 11442-11446.

[28] Pequitoy S., Aguiary A.P., Sinopoli B., Gomes D.A. Nonlinear Estimation using Mean Field Games. International conference on Network Games. Control and Optimization; (2011).

[29] Wang X.F., Chen J.F., Shi Z.G., Chen K.S. Fuzzy Control-baesd particle filter for maneuvering target tracking. Progress In Electromagnetics Research 118 (2011); 1-15.

[30] Duda R., Hart P., Stork D. Pattern classification. New York: Wiley; (2001).

[31] Zhang B., Chen M., Zhou D., Li Z. Particle-filter-based estimation and prediction of chaotic states. Chaos, Solitons and Fractals 32 (2007); 1491-1498.

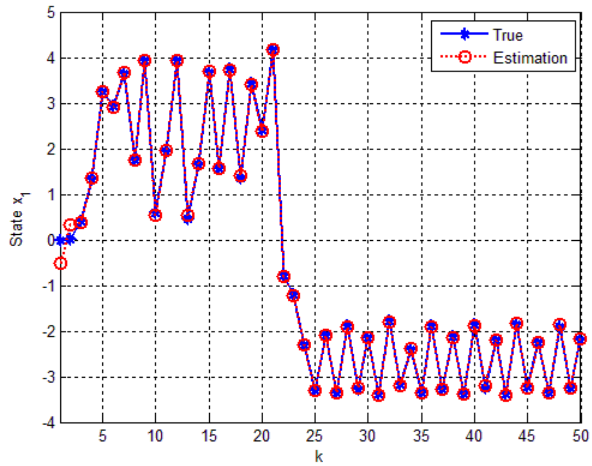

Fig. 1. The true and estimated states on $x1$.

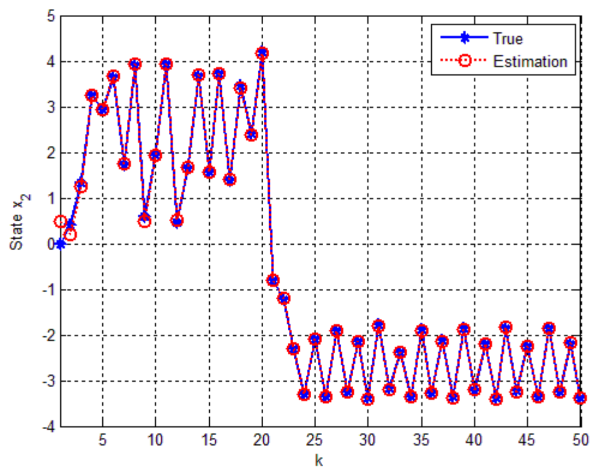

Fig. 2. The true and estimated states on $x2$.

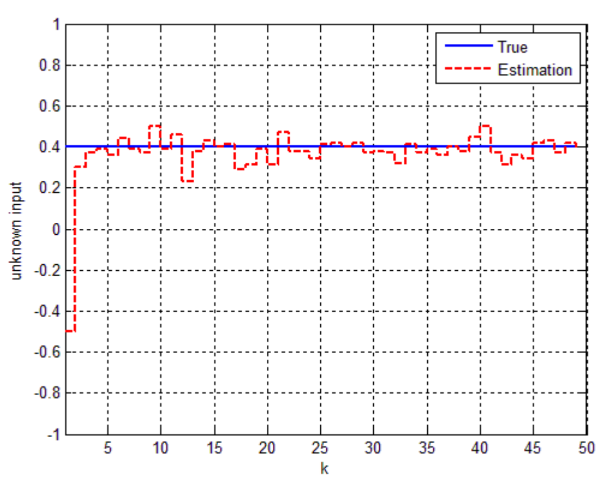

Fig. 3. The true and estimated unknown input.

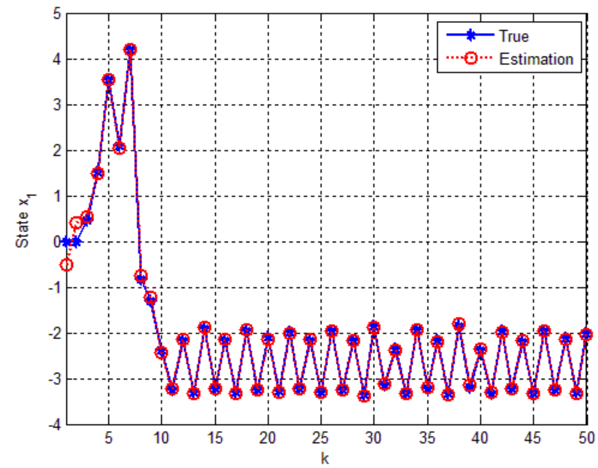

Fig. 4. The true and estimated states on $x1$ in the case of non-Gaussian noise.

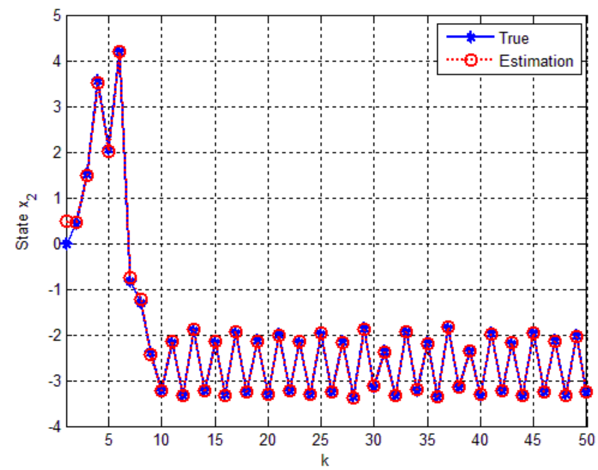

Fig. 5. The true and estimated states on $x2$ in the case of non-Gaussian noise.

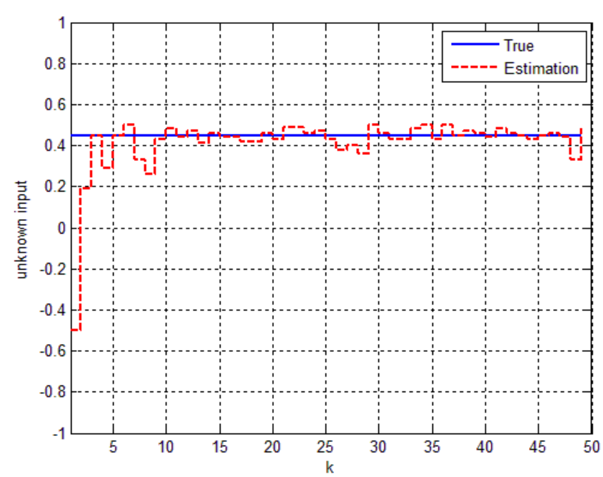

Fig. 6. The true and estimated unknown input in the case of non-Gaussian noise.